Number Oddities

The following numbers can be elegantly expressed as symmetrical equations using only the digits of the numbers:

\[ 369 = (3 \times 69) + (36 \times 9) - (3 \times 6 \times 9) \]
\[ 688 = (6 \times 88) + (68 \times 8) - (6 \times 8 \times 8) \]
\[ 639 = (6 \times 39) + (63 \times 9) - (6 \times 3 \times 9) \]

The following simple equations using three two-digit numbers remain still valid when multiplication signs are introduced between the digits of the numbers. Thus,
\[ 19 + 37 = 56 \]
\[ 18 + 39 = 57 \]
\[ 29 + 38 = 67 \]

and
\[ (1 \times 9) + (3 \times 7) = (5 \times 6) \]
\[ (1 \times 8) + (3 \times 9) = (5 \times 7) \]
\[ (2 \times 9) + (3 \times 8) = (6 \times 7) \]

The simple equation \(13^2 = 169\) is still valid when plus signs are introduced between the digits of 13 and 169, \((1 + 3)^2 = 1 + 6 + 9\) Here is a curious coincidence, \(25 \times 9^2 = 2592\)

The square number 1089 can be expressed as the difference between the squares of two reversible numbers, \(1089 = 65^2 - 56^2\)

Interestingly, it can also be expressed as the difference between two squares in two more ways, \(1089 = 55^2 - 44^2\) \(1089 = 183^2 - 190^2\)

The numbers 49 and 1680 are unique in that the addition of 1 to them as well to their halves renders them perfect squares. Thus, \(49 + 1 = 49\) \(48/2 + 1 = 25\)
\(1680 + 1 = 1681\)
\(1680/2 + 1 = 841 = 29^2\)

37 is the only two digit number which can be expressed as the difference between the sum of the squares of its digits and the product of its digits, \(37 = (3^2 + 7^2) - 3 \times 7\)

The palindromes below can be expressed as the difference between the squares of two reversible numbers, \(2772 = 96^2 - 68^2\) \(5445 = 83^2 - 38^2\) \(6336 = 80^2 - 08^2\)

Below is an interesting oddity involving the square palindrome 69696. This palindrome can be expressed as the product of two palindromes, namely, \(69696 = 6336 \times 11\)

The palindrome 6336 is interesting in that it can be represented by a palindromic expression, \(6336 = 8 \times (63 + 36) X 8\)

The square palindromes 121, 12321, 1234321, etc., are interesting. When plus signs are introduced between the digits of these numbers they still remain square, though no longer palindromic. Thus, \(1 + 2 + 1 = 4\) \(1 + 2 + 3 + 2 + 1 = 9\) \(1 + 2 + 3 + 4 + 3 + 2 + 1 = 16\).